speeds considered, the vortex generated at 4° angle of attack dissipated more rapidly than the vortex generated at 12° angle of attack.

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Determining Aircraft Stability Coefficients from Dynamic Motions

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Nomenclature

$C_{m_\alpha}(\boldsymbol{\alpha})$ = pitching moment stability coefficient, rad^{-1}
$C_{n_\beta}(\boldsymbol{\alpha})$ = yawing moment stability coefficient, rad^{-1}
$C_{m_q} + C_{m_{\dot\alpha}}(\boldsymbol{\alpha})$ = pitch damping moment stability coefficient, rad^{-1}
$C_{n_r} - C_{n_{\dot\beta}}(\boldsymbol{\alpha})$ = yaw damping moment stability coefficient, rad^{-1}
d = reference length, body diameter, ft
I_y = pitching moment of inertia, slug-ft^2
I_z = yawing moment of inertia, slug-ft^2
$M_{\delta_e}\delta_e$ = pitch trim moment, ft-lb
$N_{\delta_r}\delta_r$ = yaw trim moment, ft-lb
N = total number of data points
Q = dynamic pressure, slug/ft-sec^2
S = reference area, $S = \pi d^2/4$
$SRSQ$ = sum of residuals squared
t = time, sec
u = magnitude of velocity, fps
$\boldsymbol{\alpha}$ = total angle of attack, Eq. (3)
α = angle of attack
β = angle of sideslip

Subscript

i = quantity evaluated at time t_i

Superscript

$\hat{}$ = denotes experimental value

Introduction

THE field of atmospheric flight dynamics has been divided into two general categories: aircraft and missile dynamics. The missile dynamicist, since he was dealing with a symmetric configuration, was able to combine the equations of motion for pitching and yawing motion to obtain the Linear Aeroballistic Theory.[1-2] This theory yields an exact closed form solution for the position and orientation, as functions of time, for the symmetric missile. These solutions are then fit to a missile's position and orientation data to obtain values for the aerodynamic stability coefficients for the missile. Unfortunately, the aircraft dynamicist, dealing in most cases with asymmetric configurations, has been unable to develop exact closed form solutions for the coupled differential equations of motion of pitch and yaw. The presence of inertial coupling and lack of configurational symmetry when dealing with aircraft dynamics creates a complex situation which cannot be handled by using the technique of the missile dynamicist.

Recently, a method of dynamic analysis has been developed which is a valuable aid in studying the dynamics of aircraft.[3] The method involves an examination of the differential equations of motion of an aircraft system without the necessity of determining a solution to the equations of motion. By utilizing a forward numerical integration of the differential equation of motion, then fitting the integrated results to actual position data, the governing parameters, namely the stability coefficients, C_{m_α}, $C_{m_q} + C_{m_{\dot\alpha}}$, C_{n_β}, $C_r - C_{\dot\beta}$, can be determined as nonlinear functions of the total angle of attack of the configuration. This avoids the problem of imposing limiting assumptions on either the configuration or its motion in order to arrive at a closed form solution.

This Note investigates the application of this technique to the problem of the dynamics of aircraft type configurations. The method of numerical integration was analyzed using computer simulated two degree of angular freedom, pitch and yaw, motion for known values of the stability coefficients to determine the accuracy of the technique. Then two degree of freedom, subsonic wind-tunnel tests were conducted on an aircraft type configuration possessing digonal rotational symmetry to determine the nonlinear restoring and damping moment coefficients in pitch and yaw.

Numerical Integration Technique

Given an expression of the form,

$$\ddot\alpha = F(\alpha,\dot\alpha,t) \tag{1}$$

with a knowledge of the functionality F and the initial conditions on $\dot\alpha$ and α, it is possible to numerically integrate this equation to yield a time history of α. In dealing with the experimental dynamic motion though, usually the opposite is true. One has a time history of the dependent variable and is interested in the form of the functionality F. Models representing the form of the function F have been developed but the form and constants vary with each physical case. The numerical integration technique proposes choosing one of the various models available and then with a proper choice of initial conditions and parameters integrating the differential equation of motion Eq. (1), and comparing the integrated results with an existing set of data. This process may be repeated after correcting the parameters to improve the comparison until the integrated values fit the experimental data to a desired accuracy. Then the characteristic parameters of the chosen mathematical model have been determined for the particular set of data.

The numerical integration technique[4] was applied to a simplified form of the equations of motion for a nonrolling aircraft type configuration in which coupling takes place in the

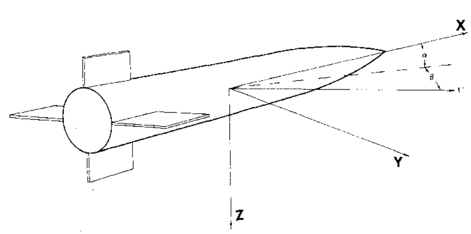

Fig. 1 Model and coordinate system.

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nonlinear aerodynamic terms, by expressing them as polynomials in total angle of attack

$$
\begin{aligned}
C_{m_\alpha}(\boldsymbol{\alpha}) &= C_{m_{\alpha 0}} + C_{m_{\alpha 2}}\boldsymbol{\alpha}^2 \\
[C_{m_q} + C_{m_{\dot\alpha}}](\boldsymbol{\alpha}) &= [C_{m_q} + C_{m_{\dot\alpha}}]_0 + [C_{m_q} + C_{m_{\dot\alpha}}]_2\boldsymbol{\alpha}^2 \\
C_{n_\beta}(\boldsymbol{\alpha}) &= C_{n_{\beta_0}} + C_{n_{\beta_2}}\boldsymbol{\alpha}^2 \\
[C_{n_r} - C_{n_{\dot\beta}}](\boldsymbol{\alpha}) &= [C_{n_r} - C_{n_{\dot\beta}}]_0 + [C_{n_r} - C_{n_{\dot\beta}}]_2\boldsymbol{\alpha}^2
\end{aligned} \tag{2}
$$

Where the total angle of attack $\boldsymbol{\alpha}$ is defined,

$$\boldsymbol{\alpha}^2 = \alpha^2 + \beta^2 \tag{3}$$

The differential equations of motion are then,

$$\ddot\alpha = \frac{QS_d{}^2}{2uI_y}[C_{m_q} + C_{m_{\dot\alpha}}](\boldsymbol{\alpha})\dot\alpha + \frac{QS_d}{I_y}C_{m_\alpha}(\boldsymbol{\alpha})\alpha + \frac{M_{\delta_e}\delta_e}{I_y} \tag{4}$$

$$\ddot\beta = \frac{QS_d{}^2}{2uI_z}[C_{n_r} - C_{n_{\dot\beta}}](\boldsymbol{\alpha})\dot\beta - \frac{QS_d}{I_z}C_{n_\beta}(\boldsymbol{\alpha})\beta + \frac{N_{\delta_r}\delta_r}{I_z} \tag{5}$$

Though these are simplified equations, they are not burdened with the constraint of symmetry of the aerodynamic terms between the pitch and yaw planes. Simultaneous forward integration using a fourth-order Runge-Kutta scheme will yield time histories of α and β for a chosen set of initial conditions and system parameters.

Equations (2) and (3) can be rewritten in a more suitable form

$$\ddot\alpha = (K_1 + K_2\boldsymbol{\alpha}^2)\dot\alpha + (K_3 + K_4\boldsymbol{\alpha}^2)\alpha + K_5 \tag{6}$$

$$\ddot\beta = (K_6 + K_7\boldsymbol{\alpha}^2)\dot\beta + (K_8 + K_9\boldsymbol{\alpha}^2)\beta + K_{10} \tag{7}$$

where the K_i's, the system parameters become the terms of interest. The success of the method depends upon the ability to determine initial values for the parameters and a method for correcting the parameter values. Using the method outlined in Ref. 4 for arriving at the values for the initial approximations, the system of Eqs. (6) and (7) can be integrated for an initial comparison with a set of angular data. In order to evaluate the accuracy of the fit a least-squares criterion is used where the residual value is the difference between the experimental value and a value computed from a series expansion about the approximate integrated value

$$SRSQ = \sum_{i=1}^{N} [(\hat\alpha_i - \alpha_i)^2 + (\hat\beta_i - \beta_i)^2] \tag{8}$$

where

$$\alpha_i = \alpha(t_i, K_j = 1,10) + \sum_{J=1}^{10} \frac{\partial\alpha}{\partial K_j}\Delta K_j \tag{9}$$

$$\beta_i = \beta(t_i, K_j = 1,10) + \sum_{J=1}^{10} \frac{\partial\beta}{\partial K_j}\Delta K_j \tag{10}$$

To arrive at the parameter corrections ΔK_j in Eqs. (9) and (10) the method of parametric differentiation was used. By defining

$$Q_j = \partial\alpha/\partial K_j, \quad R_j = \partial\beta/\partial K_j \tag{11}$$

and using Eqs. (4) and (5), a set of parametric equations was developed which could be integrated along with the equations of motion to yield the partial derivative terms in Eqs. (9) and (10) and thus the parameter corrections. This procedure was repeated until the integrated data converged to the experimental data, then the aerodynamic stability coefficients were calculated from the parameter values.

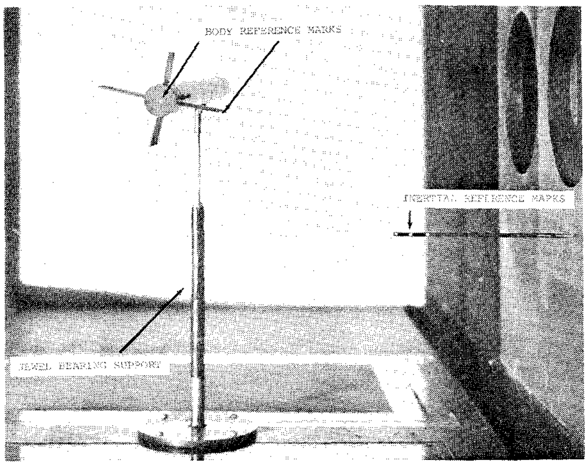

Fig. 3 Mirror view of dynamic model.

Evaluation of Fitting Technique

In order to evaluate the validity and accuracy of the numerical integration fitting technique, the differential equations of motion for pitch and yaw, Eqs. (4) and (5) were numerically integrated using a Runge-Kutta algorithm to yield time histories of α and β for the known values of the aerodynamic stability coefficients shown in Table 1. The numerical integration fitting technique was then applied to the α-β data to determine the stability coefficients which were then compared with the simulated input in Table 1. The results for the restoring moments $C_{m_{\alpha 0}}$, $C_{m_{\alpha 2}}$, $C_{n_{\beta_0}}$, and $C_{n_{\beta_2}}$ were perfect to the desired accuracy as shown in Table 1. The damping moment coefficients $[C_{m_q} + C_{m_{\dot\alpha}}]_0$, $[C_{m_q} + C_{m_{\dot\alpha}}]_2$, $[C_{n_r} - C_{n_{\dot\beta}}]_0$, and $[C_{n_r} - C_{n_{\dot\beta}}]_2$ also showed remarkable agreement. The excellent comparison of the fitted results for the coefficient values demonstrates the accuracy of the procedure when the proper mathematical model is used. The slight errors which do occur can possibly be attributed to truncation and roundoff in the numerical calculations.

Experimental Procedure

Two degree-of-freedom wind-tunnel tests were conducted on an aircraft type configuration. The configuration which was tested is representative of high-speed, low-aspect ratio

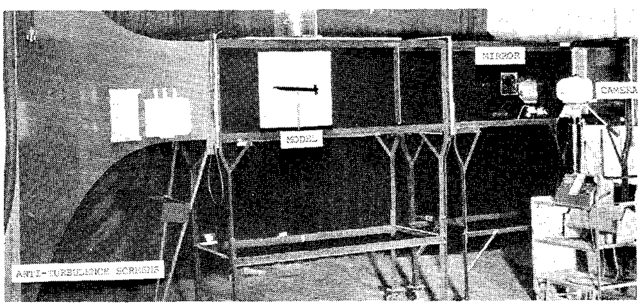

Fig. 2 Dynamic wind-tunnel testing set-up.

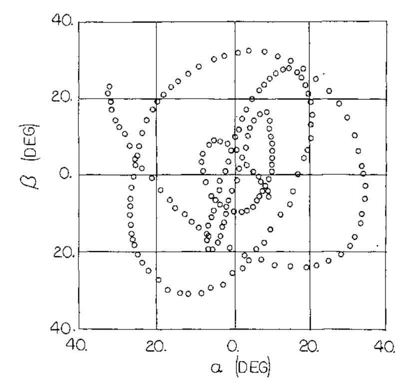

Fig. 4 Sample angular motion.

Table 1 Fitting technique evaluation

Coefficient	Simulation Input	Simulation Fit	Simulation % Difference	Experimental fit
$C_{m\alpha_0}$	-60.0	-60.0	0.	-57.2
$C_{m\alpha_2}$	0.1	0.1	0.	0.03
$[C_{m_q} + C_{m\dot{\alpha}}]_0$	-1500.0	-1500.3	0.02	-2120.0
$[C_{m_q} + C_{m\dot{\alpha}}]_2$	-2.5	-2.1	16.0	3.1
$C_{n\beta_0}$	40.0	40.0	0.	33.2
$C_{n\beta_2}$	0.05	0.05	0.	-0.01
$[C_{n_r} - C_{n\dot{\beta}}]_0$	-1500.0	-1500.7	0.04	-1620.0
$[C_{n_r} - C_{n\dot{\beta}}]_2$	7.	8.4	16.6	1.8

type aircraft. The aspect ratio of the lifting surfaces is 1.0 and 1.5 in the pitch and yaw plane, respectively, as shown in Fig. 1. The wind-tunnel support system utilized sapphire jewel bearings which have proven to minimize the effects of nonaerodynamic friction.[5] The tests were conducted at a tunnel speed of 50 fps in the open circuit low-turbulence tunnel (Fig. 2). The model was displaced to $\alpha = 40°$ then released while its motion was being recorded by a high-speed motion picture camera. The camera was focused through a mirror oriented 45° to the flow and mounted downstream of the model. Projecting the film on an optical comparator and then utilizing the reference marks shown in a sample data frame (Fig. 3), the angle of attack and angle of sideslip vs time records for the model were determined. A sample α-β motion is given in Fig. 4. This motion is characteristic of the aircraft Lissajous patterns resulting from the configuration having different frequencies in the pitch and yaw planes.

Experimental Test Results

The numerical integration fitting technique was then applied to the experimental wind-tunnel data to determine the aerodynamic stability coefficients for the test configuration. The results of the fitting procedure, Table 1, yield the expected results for this configuration. $C_{m_{\alpha 0}}$ and $[C_{m_q} + C_{m_{\dot{\alpha}}}]_0$, the terms effected by the larger lifting surfaces, were definitely larger than their respective terms in the yaw plane, $C_{n_{\beta 0}}$ and $[C_{n_r} - C_{n_{\dot{\beta}}}]_0$. The fitting procedure also yielded definite results for the nonlinear aerodynamic restoring moments $C_{m_{\alpha 2}}$, $C_{n_{\beta 2}}$ and damping moments $(C_{m_q} + C_{m_{\dot{\alpha}}}]_2$ and $[C_{n_r} - C_{n_{\dot{\beta}}}]_2$. Since the static moments were of the soft spring type, that is, decreasing stability with increasing α and β, this indicates the possibility of instability at high α, and demonstrates the importance of considering nonlinear aerodynamics in aircraft stability studies.

Conclusion

The analytical and experimental work presented in this Note has indicated that the aerodynamic stability coefficients can be extracted from the angular motion of aircraft configurations. Using the numerical integration fitting technique applied to the differential equations of motion, it is possible to determine values for the stability coefficients without imposing limiting assumptions on the configuration or its motion. The fitting technique demonstrated a high degree of accuracy as well as establishing the nonlinearity of the stability coefficients. Application of this technique is now being made to the complete pitch, yaw and roll of an aircraft configuration with primary interest in extracting the cross-coupling stability coefficients.

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Acoustics: A New Approach for Monitoring the Environment near Airports

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THE introduction of much larger aircraft to commercial and military fleets has focused attention on the need for more complete coverage of the meteorological environment at airports. Of immediate concern to those responsible for flight safety, and indeed to every pilot who may encounter one, are the intense vortices shed from the wings of large aircraft. This problem is especially critical near the ground where momentary loss of control could prove fatal. Concern over this new safety aspect has lead to a concerted effort by the FAA to learn more about the nature of vortices and their effect on other aircraft, and to a search for ways of reducing their residence time in the atmosphere or of determining their location in order that other aircraft can be advised of the potential hazard.

This effort has lead to a potpourri of suggested methods for monitoring the affected airspace, ranging from pulsed lidar to various acoustic devices. A common weakness found in most of the proposed monitoring systems is that they concentrate only on vortex detection and overlook other equally important, if less spectacular, environmental factors which affect low-level aircraft operation. For example, the effect on a planned glide slope of strong low-level wind shear or of turbulence produced by strong thermal activity can be of great importance to aircraft safety. This Note describes a relatively new device, an acoustic echo sounder, which has the potential to not only indicate the presence or absence of wake vortices, but to act as a continuous monitor of other important meteorological parameters.

In 1962, Monin[1] following contributions from many others, formulated the basic relationship describing the scatter of sound waves by turbulent wind and temperature fluctuations. More recently following a summary of this work, Little[2] outlined the potential applications of this principle which has been applied in the development of acoustic echo sounders used in remote sensing of low-level atmospheric phenomena.[2–4]

Because an acoustic wave is dependent on the medium being measured (the atmosphere) for its transmission, its

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